Math 10A HW6 Solutions

(1) $F(x) = \sin x - x\cos x$

$\Rightarrow F'(x) = \cos x - (x(-\sin x) + \cos x)$

$= \cos x + x\sin x - \cos x$

$= x\sin x$ ✓

All antiderivatives of $f(x) = x\sin x$ of the form $F(x) = \sin x - x\cos x + C$

(2) $\int x^2 dx = \frac{x^3}{3} + C = 0 + C = 3$

$\Rightarrow \frac{x^3}{3} + 3$

(3) $\int f'(x)dx = \int (2x+1)dx = x^2 + x + C$

$\Rightarrow f(x) = x^2 + x + C$

$f(0) = C = 3 \longrightarrow f(x) = x^2 + x + 3$

$\Rightarrow f(1) = 5$!

(4) $r(t) = 1 + t \quad 0 \leq t \leq 5$

$\int_0^5 (1+t)\,dt = t + \frac{t^2}{2} \Big|_0^5 = 5 + \frac{25}{2}$ liters

(5) True!

(6) True!

(7) False! Let $f(x) = x^3$.

(8) False, can recognize we're being asked to find the area of part of a circle.

(9) False, can rewrite

$$\int_{2x}^{3x} f(u)\,du = \int_{2x}^{0} f(u)\,du + \int_{0}^{3x} f(u)\,du$$

(10)

(a) $\int_{-1}^{1} x^2 - x\,dx = \frac{x^3}{3} - \frac{x^2}{2} \Big|_{-1}^{1} = \left(\frac{1}{3} - \frac{1}{2}\right) - \left(-\frac{1}{3} - \frac{1}{2}\right)$

$= \left(-\frac{1}{6}\right) + \left(+\frac{1}{6}\right)$

$= 0$

(b) $\int_{0}^{2} \sqrt{x} - 1\,dx = \frac{2x^{3/2}}{3} - x \Big|_{0}^{2} = \frac{2}{3}\sqrt{8} - 2$

(c) $\int_{1}^{2} \left(\frac{1}{x^2} + e^x\right) dx = -\frac{1}{x} + e^x \Big|_{1}^{2} = \left(-\frac{1}{2} + e^2\right) - \left(-1 + e\right)$

$= -\frac{1}{2} + e^2 + 1 - e$

$= \frac{1}{2} + e^2 - e$

(d) $\int_{0}^{1} \frac{dx}{1+x^2} = \tan^{-1}(x) \Big|_{0}^{1} = \tan^{-1}(1) - \tan^{-1}(0)$

$= \pi/4$

(e) $\int_{0}^{\pi/2} \sin(x)\,dx = -\cos x \Big|_{0}^{\pi/2} = -\cos\left(\frac{\pi}{2}\right) + \cos(0)$

$= 0 + 1 = 1$

(11) $\frac{d}{dt} \int_{1/t}^{t^3} e^{-x^2} dx = e^{-(t^3)^2} \cdot \frac{d}{dt}(t^3) - e^{-\left(\frac{1}{t}\right)^2} \cdot \frac{d}{dt}\left(\frac{1}{t}\right)$

$= e^{-t^6} \cdot 3t^2 - e^{-\frac{1}{t^2}} \cdot \left(-t^{-2}\right)$

(12) $\frac{d}{dx}\int_0^{f(x)} f(t)\,dt \underset{FTC}{=} f(u)\frac{du}{dx}$

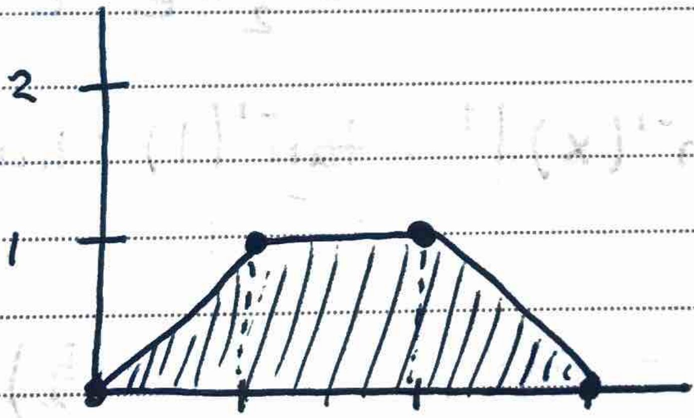
(let $u = f(x)$)

(13) False! $\frac{d}{dx}\int_a^x f(t)\,dt = f(x)$

(14) True!

(15) False! Let $f(x)=x^2$, $g(x)=x$ with $a=0$, $b=1$.

(16)



$\frac{1}{2} + 1 + \frac{1}{2} = 2$

(17) (a) $\int_0^1 f(x)\,dx = \int_0^5 f(x)\,dx - \int_1^5 f(x)\,dx$

$= 9 - 7$

$= 2$

(b) $\int_0^2 3f(x)dx = 3\int_0^2 f(x)dx = 3 \cdot 3 = 9$

(c) $\int_1^2 2f(x)+1\,dx = 2\int_1^2 f(x)dx + \int_1^2 dx$

$= 2\left(\int_0^2 f(x)dx - \left(\int_0^5 f(x)dx - \int_1^5 f(x)dx\right)\right) + 1$

$= 2(3-(9-7))+1$
$= 2(3-2)+1$
$= 3$

(18) True!
More generally if $m \le f(x) \le M$ for $a \le x \le b$ then $m(b-a) \le \int_a^b f(x)dx \le M(b-a)$.

(19) (a) $\int 2dx = 2x + C$

(b) $\int (5x+3)^6 dx = \dfrac{(5x+3)^7}{7 \cdot 5}$

(c) $\int 2t^2(t^3+1)^3 dt = \int 2t^2(t^9+3t^6+3t^3+1)dt$

$= 2\int t^{11}+3t^8+3t^5+t^2\,dt = \dfrac{2t^{12}}{12} + \dfrac{6t^9}{9} + \dfrac{6t^6}{6} + \dfrac{2t^3}{3} + C$

(d) $\int \sqrt{3x+2}\,dx$ let $u = 3x+2$

$du = 3\,dx$

$\Rightarrow \int u^{1/2} \frac{du}{3}$ $dx = \frac{1}{3} du$

$= \frac{1}{3} \int u^{1/2} du = \frac{1}{3} u^{3/2} \cdot \frac{2}{3} + C = \frac{2}{9} u^{3/2} + C$

$= \frac{2}{9} (3x+2)^{3/2} + C$

(e) $\int t^2 (t^3+7)^{-1/2} dt$ let $u = t^3+7$

$du = 3t^2 dt$

$\Rightarrow \frac{1}{3} \int u^{-1/2} du$ $\frac{1}{3} du = t^2 dt$

$= \frac{1}{3} \cdot u^{1/2} \cdot 2 = \frac{2}{3} u^{1/2} = \frac{2}{3} (t^3+7)^{1/2} + C$

(f) $\int \sin(2x-1)\,dx$ let $u = 2x-1$

$du = 2\,dx$

$\Rightarrow \frac{1}{2} \int \sin(u)\,du$ $\frac{1}{2} du = dx$

$= -\frac{1}{2} \cos u = -\frac{1}{2} \cos(2x-1) + C$

(g) $\int \sin^5(x) \cos(x)\,dx$ let $u = \sin x$

$du = \cos x\,dx$

$\Rightarrow \int u^5 du$

$= \frac{u^6}{6} = \frac{\sin^6(x)}{6} + C$

(h) $\int \frac{\sin(x)}{\cos^4(x)}\,dx = \int \left(\frac{\sin x}{\cos x}\right)\cdot\left(\frac{1}{\cos^3 x}\right)dx$

$= \int \tan(x)\sec^3(x)\,dx$ let $u = \sec x$
$du = \sec x \tan x\,dx$

$\Rightarrow \int u^2\,du$

$= \frac{u^3}{3} = \frac{\sec^3 x}{3} + C$

(i) $\int_1^9 \frac{(\sqrt{x}+1)^3}{\sqrt{x}}\,dx$

$= \int_1^9 \frac{x^{3/2} + 3x + 3x^{1/2} + 1}{x^{1/2}}\,dx$

$= \int_1^9 x + 3x^{1/2} + 3 + x^{-1/2}\,dx = \frac{x^2}{2} + \frac{2\cdot\cancel{3}x^{3/2}}{\cancel{3}} + 3x + 2x^{1/2}\Big|_1^9$

$= \left(\frac{9^2}{2} + 2\cdot 9^{3/2} + 27 + 18\right) - \left(\frac{1}{2} + 2 + 3 + 2\right)$

$= 120$

(j) $\int_0^1 (x+1)\sin(x^2+2x+1)\,dx$ let $u = x^2+2x+1$
$du = 2x+2\,dx$
$\frac{1}{2}du = x+1\,dx$

$\Rightarrow \frac{1}{2}\int_1^4 \sin(u)\,du$

$= -\frac{1}{2}\cos u\Big|_1^4 = -\frac{1}{2}(\cos 4 - \cos 1)$

(k) $\int_1^9 \left(1+\frac{1}{t}\right)^4 \cdot \frac{1}{t^2}\, dt$ let $u = 1+\frac{1}{t}$

$du = -\frac{1}{t^2} dt$

$\Rightarrow -\int_2^{10/9} u^4 du = \int_{10/9}^{2} u^4 du = \frac{u^5}{5}\Big|_{10/9}^{2}$

$= \frac{1}{5}\left(32 - \left(\frac{10}{9}\right)^5\right)$

(20) f is even $\Longleftrightarrow$ $f(-x) = f(x)$

f is odd $\Longleftrightarrow$ $f(-x) = -f(x)$

(a) $f(-x) = |-x| = |x| = f(x)$ even

(b) $f(-x) = |\cos(-x)| = |\cos(x)| = f(x)$ even

(c) $f(-x) = \sin((-x)^3) = \sin(-x^3)$

$= -\sin(x^3) = -f(x)$ odd

(d) $f(-x) = (-x)^2 \sin(-x) = -x^2 \sin(x)$

$= -f(x)$ odd

(e) $f(-x) = (-x)^2 + 1 = x^2 + 1 = f(x)$ even

(f) $f(-x) = (-x)^2 + (-x) = x^2 - x$ neither

(g) $f(-x) = \arctan(-x) = -\arctan(x)$ odd

$$\boxed{\int u\,dv = uv - \int v\,du}$$

(21) $\int_{-4}^{4} x^3 \cos(x)\,dx$ — let $f(x) = x^3\cos x$
notice $f(-x) = -x^3\cos x$
$= -f(x)$

$\Rightarrow \; = 0$

(22) (a) $\int xe^x\,dx$ — $u = x \rightarrow du = 1$
$dv = e^x \rightarrow v = e^x$

$\Rightarrow xe^x - \int e^x\,dx$

$= xe^x - e^x + C$

(b) $\int x\cos x\,dx$ — $u = x \rightarrow du = 1$
$dv = \cos x \rightarrow v = \sin x$

$\Rightarrow x\sin x - \int \sin x\,dx$

$= x\sin x + \cos x + C$

(c) $\int xe^{-4x}\,dx$ — $u = x \rightarrow du = 1$
$dv = e^{-4x} \rightarrow v = -\frac{1}{4}e^{-4x}$

$\Rightarrow -\frac{1}{4}xe^{-4x} + \frac{1}{4}\int e^{-4x}\,dx$

$= -\frac{1}{4}xe^{-4x} + \frac{1}{4}e^{-4x}\cdot\left(-\frac{1}{4}\right)$

$$\boxed{\int u\,dv = uv - \int v\,du}$$

(d) $\int e^x \sin x\,dx$

$u = e^x \Rightarrow du = e^x$
$dv = \sin x\,dx \Rightarrow v = -\cos x$

$\Rightarrow -e^x\cos x + \int e^x \cos x\,dx$

$u = e^x \Rightarrow du = e^x$
$dv = \cos x\,dx \Rightarrow v = \sin x$

$\Rightarrow = e^x\sin x - \int e^x \sin x\,dx$

$\Rightarrow \int e^x \sin x\,dx = -e^x\cos x + \int e^x\cos x\,dx$

$= -e^x\cos x + e^x\sin x - \int e^x \sin x\,dx$

$\Rightarrow 2\int e^x\sin x\,dx = e^x\sin x - e^x\cos x$

$\Rightarrow \int e^x\sin x\,dx = \frac{e^x\sin x - e^x\cos x}{2}$

(e) $\int_1^e \ln(x)\,dx$

$u = \ln(x) \Rightarrow du = dx$
$dv = 1\,dx \Rightarrow v = x$

$\Rightarrow x\ln(x)\Big|_1^e - \int_1^e x\,dx$

$= (e\ln(e) - \cancel{\ln(1)}) - \frac{x^2}{2}\Big|_1^e$

$= e - \left(\frac{e^2}{2} - \frac{1}{2}\right)$

$= e - \frac{e^2}{2} + \frac{1}{2}$

$$\boxed{\int u\,dv = uv - \int v\,du}$$

(f) $\displaystyle\int_1^e \frac{\ln(2x)}{x^2}\,dx$ $\quad u = \ln(2x) \rightarrow du = \frac{1}{x}dx$

$dv = x^{-2}dx \rightarrow v = -x^{-1}$

$$\Rightarrow -\frac{\ln(2x)}{x} + \int \frac{1}{x^2}\,dx$$

$$= \left.\frac{-\ln(2x)}{x}\right|_1^e - \left.\frac{1}{x}\right|_1^e$$

$$= -\left(\frac{\ln(2e)}{e} - \ln(2)\right) - \left(\frac{1}{e} - 1\right)$$

$$= \ln(2) - \frac{\ln(2e)}{e} + 1 - \frac{1}{e}$$